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ABSTRACT

This teacher guide is part of the materials prepared for an individualized program for ninth-grade algebra and basic mathematics students. Materials written for the program are to be used with audiovisual lessons recorded on tape cassettes. For an evaluation of the program see ED 086 545. In this guide, the teacher is provided with objectives for each topic area and guided to materials written for a given topic. Three short criterion tests are included for each topic covered. The work for this package centers on solution techniques for systems of equations in two variables and the application of these techniques for solving verbal problems. This work was prepared under an ESEA Title III contract. (JP)

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ALGEBRA I

PACKAGE 03-11

SYSTEMS OF OPEN SENTENCES IN TWO VARIABLES

Prepared by

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Under a Grant From
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SE 017 575

Many of the problems you have worked in previous packages and many that you will work in the future require that you find values of two unknown quantities. You have been expressing both unknowns in terms of one variable. However, these problems are quite often much easier to solve if you use <u>two</u> variables and <u>two</u> open sentences. Therefore, your goal when studying this package should be:

When given a verbal problem containing two undnown quantities, be able to translate it into two sentences with two variables and find the solution to the problem.

PACKAGE OBJECTIVES:

1. Given a system of simultaneous equations, determine the solution set by graphing.
2. Given a system of simultaneous equations in which a variable has a coefficient of one or a variable has the same coefficient (absolute value) in both sentences, determine the solution set by the substitution, the addition, or the subtraction method.
3. Given a system of simultaneous equations in which no variable has the same coefficient in both sentences, determine the solution set by using the addition or subtraction method.
4. Given a verbal problem containing two unknown quantities, write its solution using two sentences with two variables.
5. Given a digit problem, write its solution using two sentences with two variables.
6. Given a motion problem, write its solution using two sentences with two variables.
7. Given an age problem, write its solution using two sentences with two variables.
8. Given a system of linear inequalities, indicate their solution on the coordinate plane.

I. U. #03-11-01

SOLVING SYSTEMS OF LINEAR EQUATIONS

BY

GRAPHING

You will need to recall:

How to graph linear equations.

OBJECTIVES:

1. When asked to define "simultaneous equations" write "simultaneous equations are equations that impose two conditions on the variables at the same time.
2. When asked to explain the difference between consistent and inconsistent simultaneous equations, write "Consistent simultaneous equations have common roots, inconsistent simultaneous equations have no common roots."
3. Given a system of simultaneous equations, determine the solution set by graphing.

ACTIVITIES:

1. Study pages 415-417, SM. Be sure you understand objectives 1 and 2 and the sentence in the gray rectangle at the top of page 417. (objectives 1, 2, and 3)
2. Do some of the written exercises on page 418. Be sure to include exercises 7, 8, and 13 in those you work. (objective 3)

Criterion Test 03-11-01-01

1. Define "simultaneous equations"

2. Explain the difference between consistent and inconsistent simultaneous equations.

3. Determine the solution set by graphing:

(a) $x + y = 4$
$2x - y = 8$

(b) $3x - y = 14$
$2x + y = 6$

(c) $2x - y = 3$
$4x - 2y = -5$

Criterion Test 03-11-01-02

1. Define "simultaneous equation"

2. Explain the difference between consistent and inconsistent simultaneous equations.

3. Determine the solution set by graphing:

(a) $4x - y = 1$
$y = 2x - 3$

(b) $x - y = -4$
$y - x = 4$

(c) $x - y = 5$
$x + 5y = 11$

Criterion Test 03-11-01-03

1. Define "simultaneous equations."

2. Explain the difference between consistent and inconsistent simultaneous equations.

3. Determine the solution set by graphing.

(a) $x - 2y = 1$
$x - 3y = -2$

(b) $3x - y = 14$
$2x + y = 6$

(c) $3x + y = 11$
$2y + 6x = 22$

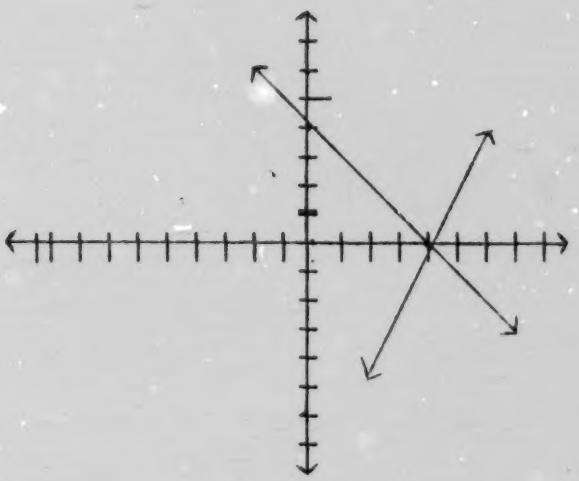
Answers to Criterion Tests

Test 03-11-01-01

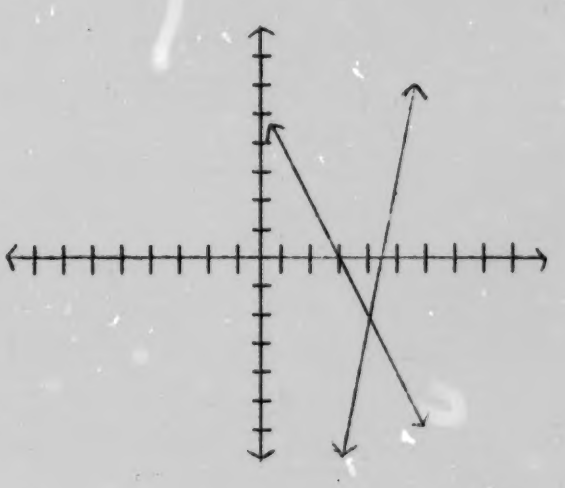
1. Simultaneous equations are equations that impose two conditions on the variable at the same time.

2. Consistent simultaneous equations have common roots, inconsistent equations have no common roots.

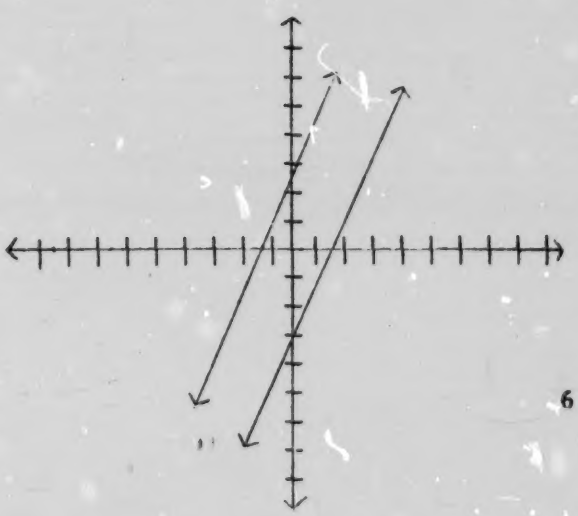
3. (a) {(4,0)}



(b) {(4,-2)}



(c) $\emptyset$



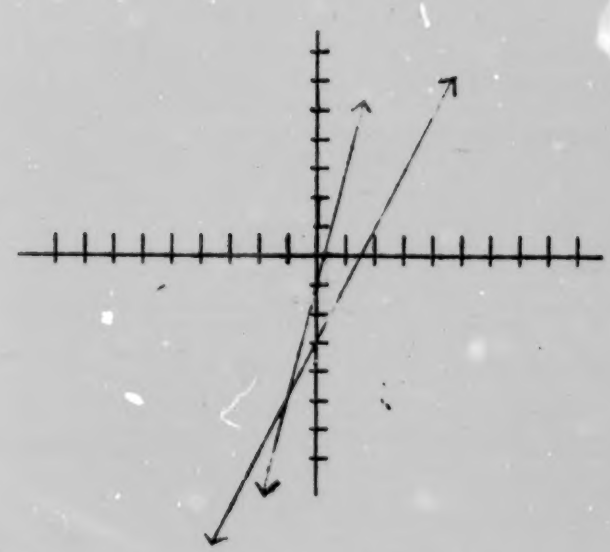
Answers to Criterion Tests

Test 03-11-01-02

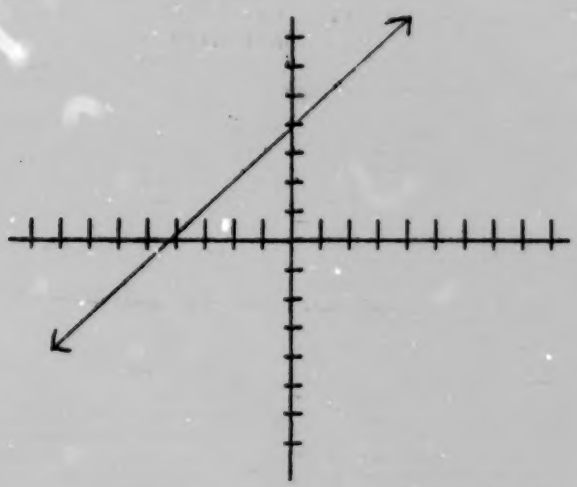
1. Simultaneous equations are equations that impose two conditions on the variables at the same time.

2. Consistent simultaneous equations have common roots, inconsistent equations have no common roots.

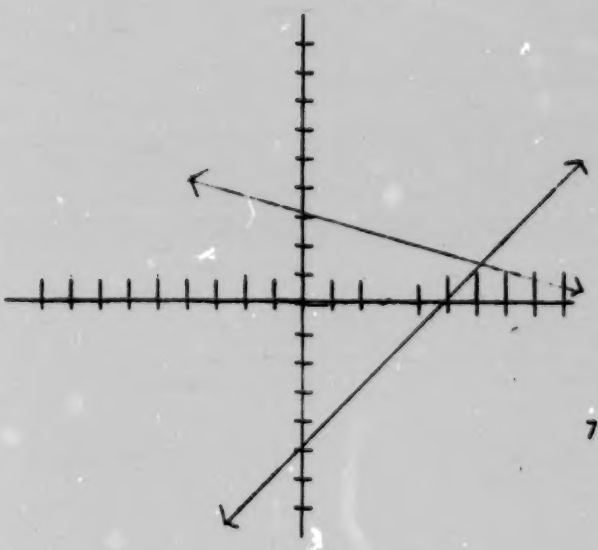
3. (a) {(-1,-5)}



(b) {all ordered pairs belonging to this line}



(c) {(6,1)}



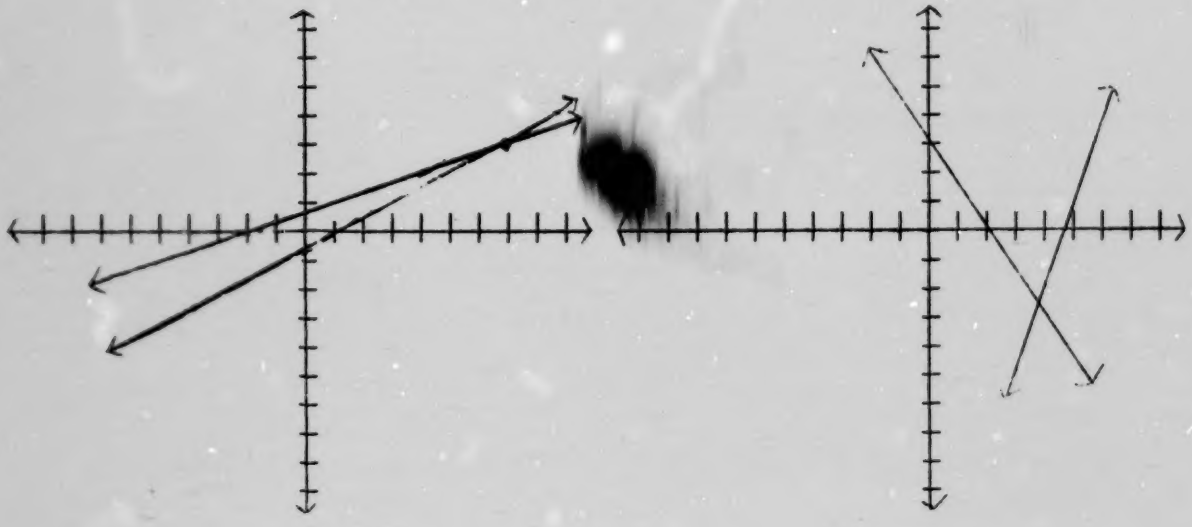
Answers to Criterion Tests

Test 03-11-01-03

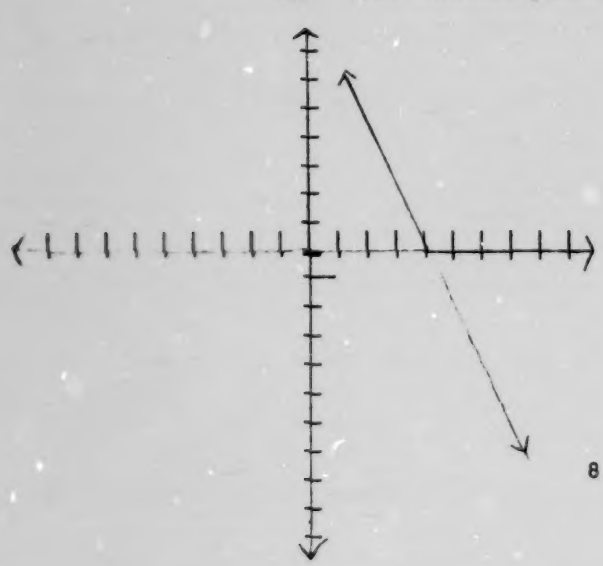
1. Simultaneous equations are equations that impose two conditions on the variables at the same time.

2. Consistent simultaneous equations have common roots, inconsistent equations have no common roots.

3. (a) {(7,3)} (b) {(4,-2)}



(c) {all ordered pairs belonging to this line}



I. U. #03-11-02

SOLVING SYSTEMS OF LINEAR EQUATIONS.

SUBSTITUTION - ADDITION - SUBTRACTION

You will need to recall:

How to solve linear equation (package #03-04)

and the meaning of "coefficient."

OBJECTIVES:

1. Given a system of simultaneous equation in which a variable has a coefficient of one, determine the solution set by the substitution method.

2. Given a system of simultaneous equations in which a variable has the same coefficient (absolute value) determine its solution set by addition or subtraction.

3. Given a system of simultaneous equations in which a variable has a coefficient of one or a variable has the same coefficient (absolute value) in both senterces, determine the solution set by the substitution, the addition, or the subtraction method.

ACTIVITIES:

1. Study page 426 S & H (objectives 1 and 3)

2. Study pages 419-420, S & H (objectives 2 and 3)

Exercises: Do some of exercises on page 426, do some of exercises page 420. You may want to try some of the part B exercises on page 421 because some of the age problems you will work in instructional unit 03-11-07 result in equations such as these.

Criterion Test 03-11-02-01

1. Solve by substitution:

(a) $x = 8y$
$x - 4y = 12$

(b) $x + 3y = 5$
$-3x + 2y = 18$

2. Solve by addition or subtraction:

(a) $3x + y = 9$
$2x - y = 1$

(b) $2x + 3y = -5$
$5x + 3y = 1$

3. Solve by substitution, addition, or subtraction:

(a) $x + 3y = 14$
$x - 2y = -1$

(b) $4x + 3y = 7$
$4x + 4y = 12$

Criterion Test 03-11-02-02

1. Solve by substitution:

(a) $x = 3y$
$2x - 5y = 4$

(b) $y - 2x = 0$
$x + 2y = 9$

2. Solve by addition or subtraction:

(a) $3x - 2y = -9$
$3x - 5y = -12$

(b) $5x - y = -23$
$-3x + y = 15$

3. Solve by substitution, addition, or subtraction:

(a) $2x + y = 7$
$2x - y = 5$

(b) $14x - 4y = -16$
$9x - 4y = -19$

Criterion Test 03-11-02-03

1. Solve by substitution:

(a) $x - 2y = -8$
$3x + y = -3$

(b) $x = y - 4$
$3y + 2x = 12$

2. Solve by addition or subtraction:

(a) $3x - 2y = 1$
$4x + 2y = 6$

(b) $7x - y = -22$
$7x + 2y = 2$

3. Solve by substitution, addition, or subtraction:

(a) $2x - 3y = -4$
$y + x = -2$

(b) $3x - 5y = -12$
$2x + 5y = 17$

Answers to Criterion Tests

Test 03-11-02-01

1. (a) $x = 8y$
$x - 4y = 12$
$(8y) - 4y = 12$
$y = 3$
$x = 8 \cdot 3 = 24$
$\{(24,3)\}$

(b) $x + 3y = 5$
$-3x + 2y = 18$
$x = 5 - 3y$
$-3(5 - 3y) + 2y = 18$
$11y = 33$
$y = 3$
$x = 5 - 3(3) = -4$
$\{(-4,3)\}$

2. (a) $3x + y = 9$
$2x - y = 1$
$5x = 10$
$x = 2$
$3(2) + y = 9$
$y = 3$
$\{(2,3)\}$

(b) $2x + 3y = -5$
$5x + 3y = 1$
$-3x = -6$
$x = 2$
$2(2) + 3y = -5$
$y = -3$
$\{(2,-3)\}$

3. (a) $\{(5,3)\}$

(b) $\{(-2,5)\}$

Test 03-11-02-02

1. (a) $x = 3y$
$2x - 5y = 4$
$2(3y) - 5y = 4$
$y = 4$
$x = 3 \cdot 4 = 12$
$\{(12,4)\}$

(b) $y - 2x = 0$
$x + 2y = 9$
$y = 2x$
$x + 2(2x) = 9$
$5x = 9$
$x = \frac{9}{5}$

$y = 2(\frac{9}{5}) = \frac{18}{5}$ $\{(\frac{9}{5}, \frac{18}{5})\}$

2. (a) $3x - 2y = -9$
$3x - 5y = -12$
$3y = 3$
$y = 1$
$3x - 2(1) = -9$
$x = -\frac{7}{3}$

$\{(-\frac{7}{3}, 1)\}$

(b) $5x - y = -23$
$-3x + y = 15$
$2x = -8$
$x = -4$
$5(-4) - y = -23$
$y = 3$
$\{(-4,3)\}$

3. (a) $\{(3,1)\}$

(b) $\{(\frac{3}{5}, \frac{61}{10})\}$

Answers to Criterion Tests

Test 03-11-02-03

1. (a) $x - 2y = -8$
$3x + y = -3$
$x = 2y - 8$
$3(2y - 8) + y = -3$
$7y = 21$
$y = 3$
$x = 2(3) - 8 = -2$
$\{(-2,3)\}$

(b) $x = y - 4$
$3y + 2x = 12$
$3y + 2(y - 4) = 12$
$5y = 20$
$y = 4$
$x = 4 - 4 = 0$
$\{(0,4)\}$

2. (a) $3x - 2y = 1$
$4x + 2y = 6$
$7x \quad = 7$
$x = 1$
$3(1) - 2y = 1$
$y = 1$
$\{(1,1)\}$

(b) $7x - y = -22$
$7x + 2y = 2$
$-3y = -24$
$y = 8$
$7x - 8 = -22$
$x = -2$
$\{(-2,8)\}$

3. (a) $\{(-2,0)\}$

(b) $\{(1,3)\}$

I. U. #03-11-03

SOLVING SYSTEMS OF LINEAR EQUATIONS-

UNLIKE COEFFICIENTS

OBJECTIVES:

1. Given a system of simultaneous equations in which no variable has the same coefficient in both sentences, determine the solution set by using the addition or subtraction method.

ACTIVITIES:

1. Study page 423, S & M. (Objective 1)

   You should do most of the exercises on page 424, S & M.

Criterion Test 03-11-03-01

1. Determine the solution set:

(a) $8x + 6y = 10$
$-4x + 3y = -1$

(b) $2x - 3y = -14$
$3x + 7y = 48$

(c) $\frac{x}{2} - 2y = 0$

$\frac{x}{6} - \frac{y}{3} = 1$

Criterion Test 03-11-03-02

1. Determine the solution set:

(a) $9x - 2y = 2\frac{1}{2}$
$5x - 6y = -3\frac{1}{2}$

(b) $2x + 7y = 52$
$-3x + 5y = -16$

(c) $3x - \frac{y}{2} = 10$

$\frac{2x}{3} - y = 6$

Criterion Test 03-11-03-03

1. Determine the solution set:

(a) $5x + 9y = 6$
$6x + 5y = 13$

(b) $4x + 3y = -2$
$8x - 2y = 12$

(c) $2x - \frac{3y}{2} = 10$

$\frac{3x}{10} - \frac{y}{20} = \frac{4}{5}$

Answers to Criterion Tests

Test 03-11-03-01

1. (a) $\{(\frac{3}{4}, \frac{2}{3})\}$ (b) $\{(2, 6)\}$ (c) $\{(12, 3)\}$

Test 03-11-03-02

1. (a) $\{(\frac{1}{2}, 1)\}$ (b) $\{(12, 4)\}$ (c) $\{(6, -2)\}$

Test 03-11-03-03

1. (a) $\{(3, -1)\}$ (b) $\{(1, -2)\}$ (c) $\{(2, -4)\}$

I. U. #03-11-04

SOLVING VERBAL PROBLEMS

USING

SYSTEMS OF EQUATIONS

OBJECTIVE:

1. Given a verbal problem containing two unknown quantities, write its solution using two variables.

ACTIVITIES:

1. Study the example on page 421, S & M. Be sure you read a problem carefully before setting out to solve it. Understand the situation described then be sure to use the 4 steps given in the example. Finally, write the answer to the problem.

Exercises: (Be sure to use two variables in the solution of these problems) Do some problems from the following pages in S & M: 422, 423, 425, and 427.

Criterion Test 03-11-04-01

1. Write the solution.

   (a) The difference between two times one number and a smaller one is 39. The sum of the smaller and three times the larger is 96. Find the number.

   (b) A rectangular room is 3 feet longer than it is wide and has a perimeter of 50 feet. What are the dimensions of the room?

   (c) In a math class there are 3 fewer girls than twice the number of boys. If there were 3 more boys in the class, there would be an equal number of boys and girls. How many girls are in the class?

Criterion Test 03-11-04-02

1. Write the solution.

   (a) Lotta Cash invested $1400, some at 6% and some at 5%. Her total annual income from her investment is $73. How much does she have invested at each rate?

   (b) Twice a number is seven more than a larger number. Three times the smaller is one more than twice the larger. Name the two numbers.

   (c) A 38 foot rope is cut into two pieces so that one piece is 4 feet less than twice the other. What are the lengths of the pieces?

Criterion Test 03-11-04-03

1. Write the solution.

   (a) Mrs. Proud's baby has four less teeth than Mrs. Prouder's baby. Together the babies have 18 teeth. How many teeth does Mrs. Prouder's baby have?

   (b) Marvin's geometry grade was ten points less than his history grade. His geometry grade was also $\frac{7}{8}$ of his history grade. What were his grades?

   (c) Harry Heap has 27 coins in dimes and quarters. Altogether he has $4.05. How many dimes does he have?

Answers to Criterion Tests

Test 03-11-04-01

1. (a) x = smaller number
y = larger number
$2y - x = 39$
$x + 3y = 96$

{(15, 27)}
the numbers are 15 and 27

(b) x = length
y = width
$x - y = 3$
$2x + 2y = 50$

{(14, 11)}
the room is 14 feet by 11 feet

(c) x = boys
y = girls
$2x - y = 3$
$x + 3 = y$

{(6, 9)}
there are nine girls in the class

Test 03-11-02

1. (a) x = amount invested at 5%
y = amount invested at 6%
x + y = 1400
.05x + .06y = 73

{(1100, 300)}
She had $1100 invested at 5% and $300 invested at 6%

(b) x = smaller number
y = larger number
2x = 7 + y
3x = 1 + 2y

{(13, 19)}
the numbers are 13 and 19

(c) x = one piece
y = other piece
x + y = 38
x = 2y - 4

{(24, 14)}
the piece are 24 feet and 14 feet in length

Test 03-11-04-03

1. (a) x = number of teeth Mrs. Proud's baby has
y = number of teeth Mrs. Prouder's baby has
$y - x = 4$
$y + x = 18$

{(7, 11)}
Mrs. Prouder's baby has 11 teeth

(b) x = geometry grade
y = history grade
$y - 10 = x$
$x = \frac{7}{8}y$

{(70, 80)}
geometry grade was 70, history grade was 80

(c) x = dimes
y = quarters
$x + y = 27$
$.10x + .25y = 4.05$

{(18, 9)}
He has 18 dimes

I. U. #03-11-05

DIGIT PROBLEMS

You will need to recall:

That the expanded form of a two-digit standard numeral such as 37 is 3 x 10 + 7.

Although digit problems may seem to have little practical value, they will give you valuable experience in translating from verbal problems to mathematical sentences. Maybe digit problems should be thought of as challenging puzzles!

OBJECTIVES:

1. When asked to represent a two digit numeral with variables for its digits, write 10t + u.
2. When asked to write the numeral 10t + u with its digits reversed, write 10u + t.
3. When asked to express the sum of the digits of a two-digit numeral in terms of variables, write t + u.
4. Given a digit problem, write its solution using two sentences with two variables.

ACTIVITIES:

1. Study page 428 in S & M. (Objectives 1, 2, 3, 4)
2. Do some of the part A problems on page 429, S & M. (Objective 4)

Criterion Test 03-11-05-01

1. Represent a two-digit numeral with variables for its digits.

2. Write the numeral 10t + u with its digits reversed.

3. Express the sum of the digits of a two-digit numeral in terms of variables.

4. Write a solution:

   (a) The sum of the digits of a two-digit numeral is 11. The tens digit is 3 less than the units digit. Find the number.

   (b) The sum of the digits of a two-digit numeral is 10. The value of the number with its digits reversed is 72 more than the number. Find the number.

Criterion Test 03-11-05-02

1. Represent a two-digit numeral with variables for its digits.

2. Write the numeral 10t + u with its digits reversed.

3. Express the sum of the digits of a two-digit numeral in terms of variables.

4. Write a solution:

   (a) The units digit of a two-digit numeral is four more than the tens digit. If the digits are reversed the resulting number is one less than twice the original number. Find the number.

   (b) The sum of the digits of a two digit numeral is 8. The number with the digits reversed is eleven times the original tens digit. Find the number.

Criterion Test 03-11-05-03

1. Represent a two-digit numeral with variables for its digits.

2. Write the numeral 10t + u with its digits reversed.

3. Express the sum of the digits of a two-digit numeral in terms of variables.

4. Write a solution:

   (a) The sum of the digits of a two-digit numeral is 5. The number with its digits reversed is sixteen times the original tens digit. Find the number.

   (b) The units digit of a two-digit numeral exceeds three times the tens digit by three. Reversing the digits increased the number by 63. Find the number.

Answers to Criterion Tests

Test 03-11-05-01

1. $10t + u$

2. $10u + t$

3. $t + u$

4. (a)
t = tens digit
u = units digit
$t + u = 11$
$t = u - 3$
$(u - 3) + u = 11$
$u = 7$
$t = 4$
the number is 47

(b)
t = tens digit
u = units digit
$t + u = 10$
$10u + t = 72 + 10t + u$
$9u - 9t = 72$
$9u + 9t = 90$
$18u = 162$
$u = 9$
$t = 1$
The number is 19.

Test 03-11-05-02

1. $10t + u$

2. $10u + t$

3. $t + u$

4. (a)
t = tens digit
u = units digit
$u = 4 + t$
$10u + t = 2(10t + u) - 1$
$10u + t = 20t + 2u - 1$
$8u - 19t = -1$
$8(4 + t) - 19t = -1$
$-11t = -33$
$t = 3$
$u = 7$
the number is 37

(b)
t = tens digit
u = units digit
$t + u = 8$
$10u + t = 11t$
$10u - 10t = 0$
$10u + 10t = 80$
$20u = 80$
$u = 4$
$t = 4$
the number is 44

Answers to Criterion Tests (Cont.)

Test 03-11-05-03

1. $10t + u$

2. $10u + t$

3. $t + u$

4. (a)
t = tens digit
u = units digit
$t + u = 5$
$10u + t = 16t$
$10u - 15t = 0$
$10u + 10t = 50$
$-25t = -50$
$t = 2$
$u = 3$
the number is 23

(b)
t = tens digit
u = units digit
$u = 3t + 3$
$10u + t - 63 = 10t + u$
$9u - 9t = 63$
$u - t = 7$
$(3t + 3) - t = 7$
$2t = 4$
$t = 2$
$u = 9$
the number is 29

I. U. #03-11-06

MOTION PROBLEMS

You will need to recall:

the distance formula, d = rt.

distance = rate x time

OBJECTIVES:

1. Given a motion problem, write a solution using two sentences with two variables.

ACTIVITIES:

1. Study example page 430, S & M. You should work all of part A and some of part B problems pp. 430-431, S & M.

Criterion Test 03-11-06-01

1. Write a solution.

   (a) Joe can row 12 miles upstream in 4 hours. He can row 28 miles downstream in the same time. How fast can he row in still water.

   (b) It took $4\frac{2}{3}$ hours for a plane to make a 1400 mile trip against the wind. It took on 4 hours for the return trip with the same wind. What is the rate of the wind.

Criterion Test 03-11-06-02

1. Write a solution.

   (a) Jim can row twice as far down stream as he can upstream in the same time. If he can row 20 miles upstream in 8 hours, what is the speed of the current?

   (b) A boat traveled 108 miles upstream in 9 hours. The return trip took 6 hours. What is the boat's speed in still water?

Criterion Test 03-11-06-03

1. Write a solution.

   (a) It takes Kerri the same amount of time to go 30 miles upstream as it does to go 40 miles downstream. It takes her 15 minutes to go 3 miles upstream. What is the rate of the current?

   (b) A motorboat goes 30 miles downstream in $1\frac{1}{4}$ hours. The return trip against the current takes 3 hours. Find the boat's speed in still water.

Answers to Criterion Tests

Test 03-11-06-01

1. (a) x = rate in still water.
y = rate of current

	d =	r ·	t
upstream	12	x - y	4
downstream	28	x + y	4

$4(x - y) = 12$
$4(x + y) = 28$
$4x - 4y = 12$
$4x + 4y = 28$
$8x = 40$
$x = 5$

he can row 5 m.p.h. in still water

(b) x = speed of plane in still air
y = speed of wind

	d =	r ·	t
with	1400	x + y	4
against	1400	x - y	$4\frac{2}{3}$

$4(x + y) = 1400$

$4\frac{2}{3}(x - y) = 1400$

$4x + 4y = 1400 \longrightarrow x + y = 350$
$14(x - y) = 4200 \longrightarrow x - y = 300$
$14x - 14y = 4200 \longrightarrow 2y = 50$
$y = 25$

the speed of the wind is 25 m.p.h.

Answers to Criterion Tests (Cont.)

Test 03-11-06-02

1. (a) x = Jim's rate in still water
y = rate of current

	d =	r ·	t
upstream	20	x - y	8
downstream	40	x + y	8

$8(x - y) = 20$
$8(x + y) = 40$
$8x - 8y = 20$
$8x + 8y = 40$
$-16y = -20$
$y = 1\frac{1}{4}$

the speed of the current is $1\frac{1}{4}$ m.p.h.

(b) x = boats speed in still water
y = speed of current

	d =	r ·	t
upstream	108	x - y	9
downstream	108	x + y	6

$9(x - y) = 108$
$6(x + y) = 108$
$9x - 9y = 108 \longrightarrow x - y = 12$
$6x + 6y = 108 \longrightarrow x + y = 18$
$2x = 30$
$x = 15$

the boat's speed in still water is 15 m.p.h.

Test 03-11-06-03

1. (a) x = speed of boat in still water
y = speed of current

	d =	r ·	t
upstream	30	x - y	2½
downstream	40	x + y	2½

3 mi. in 15 min. =
30 mi. in 150 min.
150 min. = 2½ hrs.

$2\frac{1}{2}(x - y) = 30 \longrightarrow 5(x - y) = 60$
$2\frac{1}{2}(x + y) = 40 \longrightarrow 5(x + y) = 80$
$x - y = 12$
$x + y = 16$
$-2y = -4$
$y = 2$

the speed of the current is 2 m.p.h.

(b) x = boat's speed in still water
y = rate of current

	d =	d ·	t
upstream	30	x - y	3
downstream	30	x + y	1½

$3(x - y) = 30$
$1\frac{1}{2}(x + y) = 30$

$x - y = 10$
$x + y = 20$
$2x = 30$
$x = 15$

the boat's speed in still water is 15 m.p.h.

I. U. # 03-11-07

AGE PROBLEMS

OBJECTIVES:

1. Given an age problem, write its solution using two sentences and two variables.

ACTIVITIES:

1. Study the example page 432, S & M. Work most of part A problems and some of part B, pp. 432, 433 S & M.

Criterion Test 03-11-07-01

1. Write a solution.

   (a) Jackie's mother is three times as old as Jackie. In 10 years Jackie will be as old as her mother was 20 years ago. Find their present ages.

   (b) Jeannie is $\frac{3}{4}$ as old as Judy. In 7 years she will be $\frac{4}{5}$ as old as Judy. How old is Judy?

Criterion Test 03-11-07-02

1. Write a solution.

   (a) Six years ago, Mr. Miller was two years more than five times as old as his son. In another six years, he will be eleven years more than twice as old as his son. How old is he?

   (b) A man is now 5 times as old as his son. In five years he will be only 3 times as old as his son will be then. How old is the son now?

Criterion Test 03-11-07-03

1. Write a solution.

   (a) Three years ago, Jack was one year more than twice as old as Jill. Six years from now, he will be 10 years more than half her age. How old is Jack?

   (b) Mrs. Happy is $\frac{6}{7}$ as old as Mr. Happy. Four years ago, when they were married, she was $\frac{5}{6}$ as old as he. How old is each?

Answers to Criterion Tests

Test 03-11-07-01

1. (a)

Time	Jackie	Mother
now	x	y
in 10 yrs	x + 10	y + 10
20 yrs ago	x - 20	y - 20

$$y = 3x$$
$$x + 10 = y - 20$$
$$x + 10 = (3x) - 20$$
$$-2x = -30$$
$$x = 15$$
$$y = 45$$

Jackie is 15, her mother is 45.

(b)

Time	Jeannie	Judy
Now	x	y
In 7 yrs	x + 7	y + 7

$$x = \frac{3}{4}y$$

$$x + 7 = \frac{4}{5}(y + 7)$$

$$5x + 35 = 4y + 28$$

$$5(\frac{3}{4}y) + 35 = 4y + 28$$

$$15y + 140 = 16y + 112$$
$$-y = -28$$
$$y = 28$$

Judy is 28 years old.

Answers to Criterion Tests (Cont) TEST 03-11-07-02

1. (a)

Time	Mr. Miller	Son
Now	x	y
6 yrs ago	x - 6	y - 6
In 6 yrs	x + 6	y + 6

$$x - 6 = 2 + 5(y - 6) \longrightarrow x - 5y = -22$$
$$x + 6 = 11 + 2(y + 6) \longrightarrow x - 2y = 17$$
$$-3y = 39$$
$$y = 13$$
$$x = 43$$

Mr. Miller is 43.

(b)

Time	Man	Son
Now	x	y
In 5 yrs	x + 5	y + 5

$$x = 5y$$
$$x + 5 = 3(y + 5)$$
$$(5y) + 5 = 3y + 15$$
$$2y = 10$$
$$y = 5$$

The son is 5 years old

Answers to Criterion Test Criterion Test 03-11-07-03

1. (a)

Time	Jack	Jill
Now	x	y
3 yrs ago	x - 3	y - 3
6 yrs from now	x + 6	y + 6

$x - 3 = 1 + 2(y - 3) \longrightarrow x - 2y = -2$

$x + 6 = 10 + \frac{1}{2}(y + 6)$

$x - \frac{1}{2}y = 7 \longrightarrow 4x - 2y = 28$

$-3x = -30$

$x = 10$

Jack is 10 years old

(b)

Time	Mrs. Happy	Mr. Happy
Now	x	x
4 yrs ago	x - 4	y - 4

$$x = \frac{6}{7}y$$

$$x - 4 = \frac{5}{6}(y - 4)$$

$$(\frac{6}{7}y) - 4 = \frac{5}{6}(y - 4)$$

$36y - 168 = 35y - 140$

$y = 28$

$x = 24$

Mrs. Happy is 24 and Mr. Happy is 28.

I. U. #03-11-08

SYSTEMS OF LINEAR INEQUALITIES

You will need to recall:

how to graph a linear equation.

OBJECTIVES:

1. When asked to explain the terms *half-plane* and *boundary*, write *Any line separates the plane into two regions called half-planes. The line is the boundary of each half-plane.*

2. When asked to define open half-plane, write *An open half-plane is a half-plane without its boundary.*

3. When asked to define a closed half-plane, write *A closed half-plane is the union of a half-plane and its boundary.*

4. Given a linear inequality, determine if its graph is an open half-plane or a closed half-plane.

5. Given a linear inequality, draw its graph on the coordinate plane.

6. Given a system of linear inequalities, indicate their solution set on the coordinate plane.

ACTIVITIES:

1. Study page 435, S & M (Objectives 1, 2, 3, and 4)
2. Study page 436, S & M (Objective 5)
3. Study pages 437 and 438, S & M (Objective 6)
4. Do some part A exercises page 437, S & M (Objective 5)
5. Do some part A exercises page 438, S & M (Objective 6)

Criterion Test 03-11-08-01

1. Explain the terms *half-plane* and *boundary*.

2. Define *open half-plane*.

3. Define *closed half-plane*.

4. Indicate whether the graph of each inequality is a closed half-plane or an open half-plane.

   (a) $y \geq x + 3$ (c) $y > 3$

   (b) $2x - y \leq 5 + x$ (d) $x - 12 < 13 - y$

5. Graph in the coordinate plane.

   (a) $y \geq x + 3$ (b) $y > 3$

6. Graph this system:

   $y > 2x - 6$
   $y < 3x + 4$

Criterion Test 03-11-08-02

1. Explain the terms *half-plane* and *boundary*.

2. Define *open half-plane*.

3. Define *closed half-plane*.

4. Indicate whether the graph of each inequality is a closed half-plane or an open half-plane.

   (a) $y \leq 2 - x$ (c) $x \geq 1$

   (b) $2y + x < 5$ (d) $x + y > 2$

5. Graph in the coordinate plane.

   (a) $y \leq 2 - x$ (b) $x > 1$

6. Graph this system.

   $2x + y > 1$
   $2x + 3y \geq 6$

Criterion Test 03-11-08-03

1. Explain the terms *half-plane* and *boundary*.

2. Define *open half-plane*.

3. Define *closed half-plane*.

4. Indicate whether the graph of each inequality is a closed half-plane or an open half-plane.

   (a) $x + y \leq 3$ (c) $y < 3x$

   (b) $2x + y > 1$ (d) $y \leq \frac{-x}{3}$

5. Graph in the coordinate plane.

   (a) $x + y \leq 3$ (b) $y \leq 3x$

6. Graph this system.

   $x + 3y \geq 2$

   $2x + 3y \leq 7$

Answers to Criterion Tests

Test 03-11-08-01

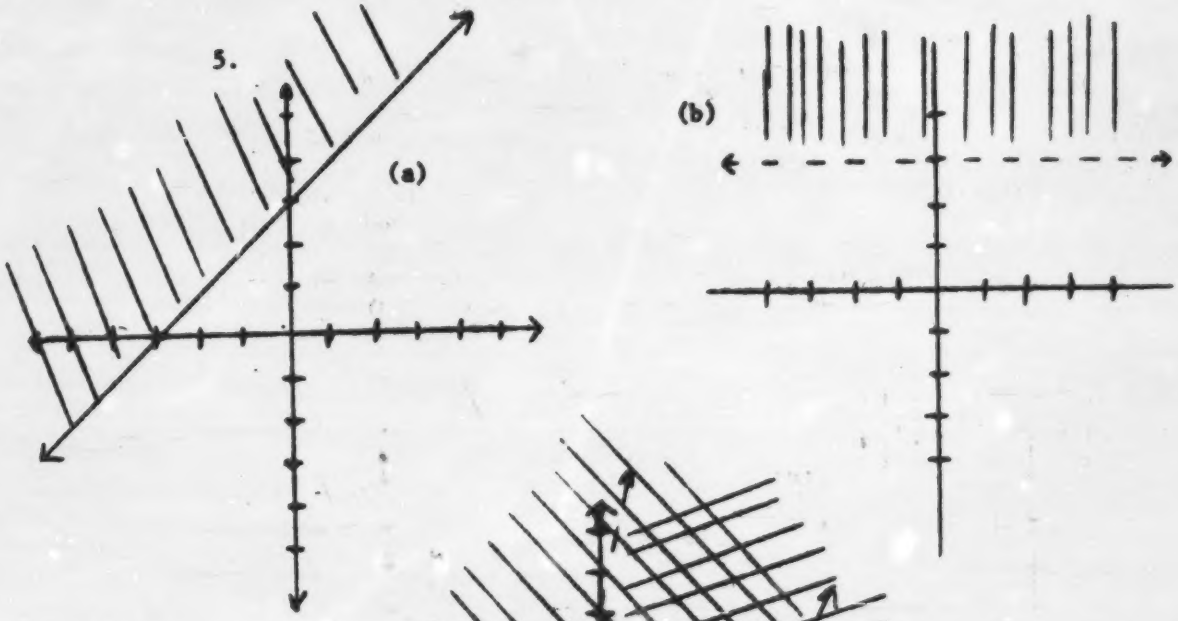
1. Any line separates the plane into two regions called half-planes. The line is the boundary of each half-plane.

2. An open half-plane is a half-plane without its boundary.

3. A closed half-plane is the union of a half-plane and its boundary.

4. (a) closed
   (b) closed
   (c) open
   (d) open

5.



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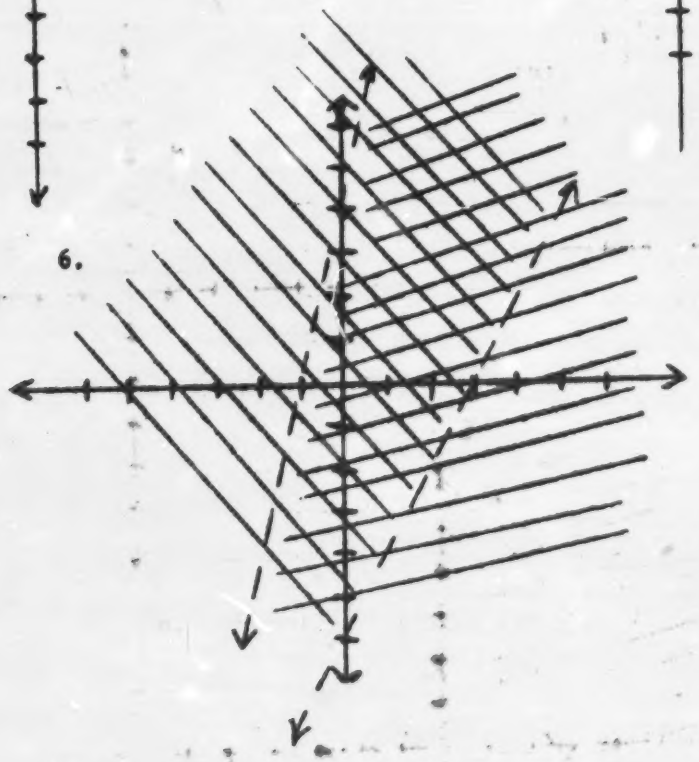


Answers to Criterion Tests (Cont.) Test 03-11-08-02

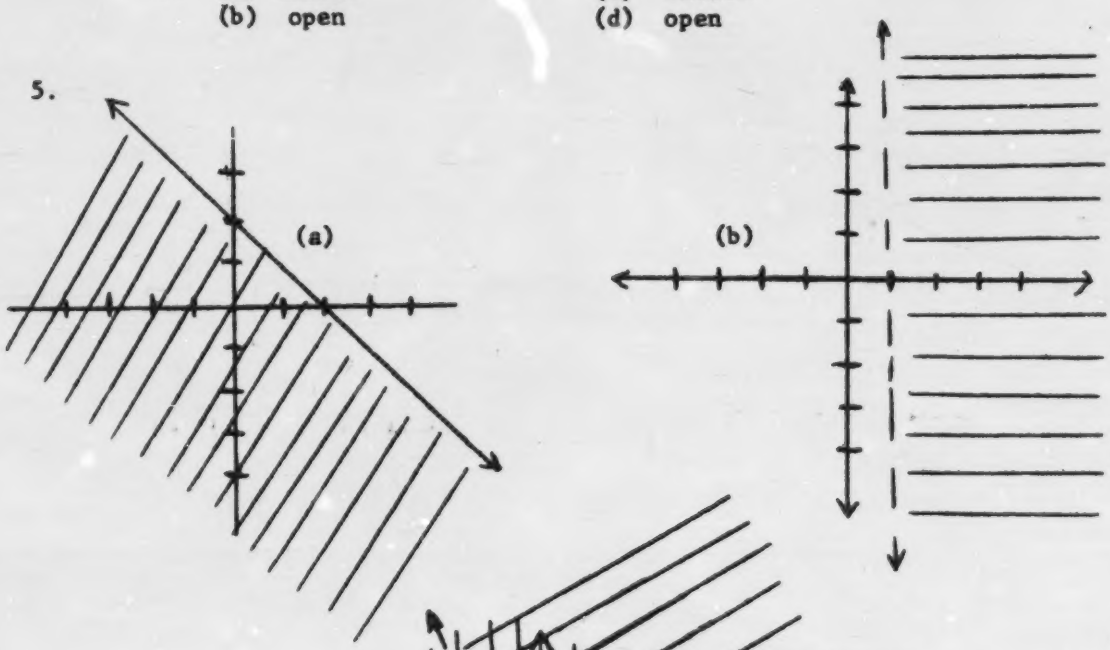
1. Any line separates the plane into two regions called half-planes. The line is the boundary of each half-plane.

2. An open half-plane is a half-plane without its boundary.

3. A closed half-plane is the union of a half-plane and its boundary.

4. (a) closed
   (b) open
   (c) closed
   (d) open

5.



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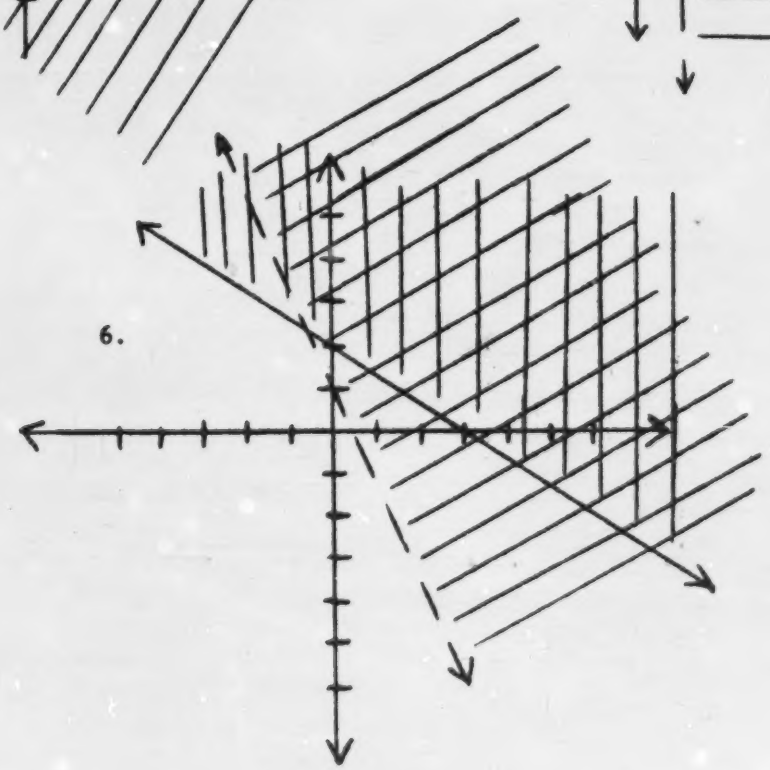


Criterion Test 03-11-08-03

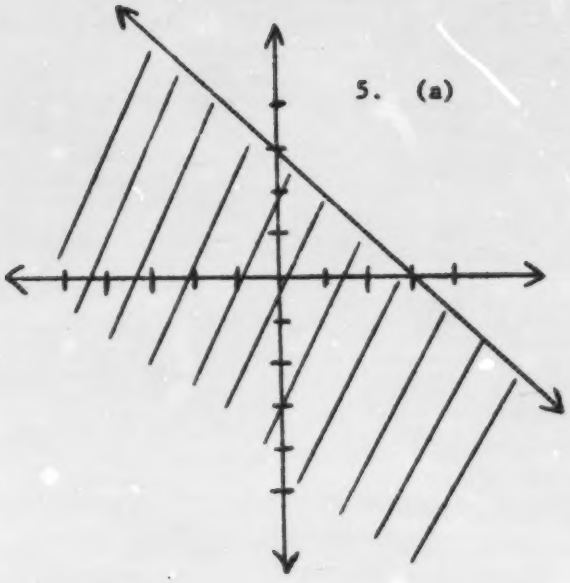
1. Any line separates the plane into two regions called half-planes. The line is the boundary of each half-plane.

2. An open helf-plane is a half-plane without its boundary.

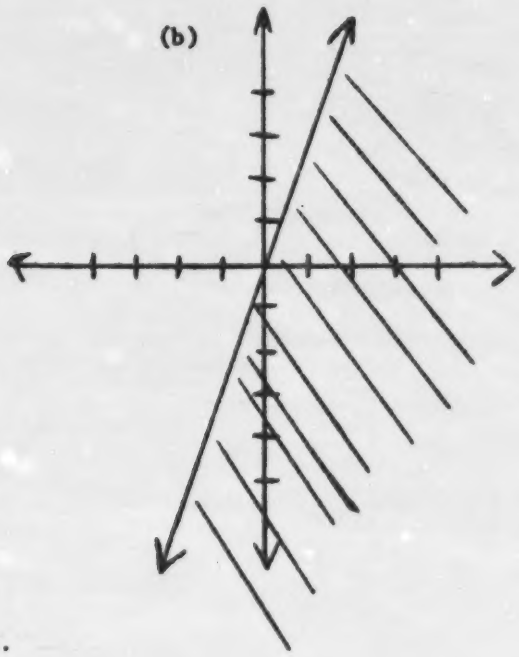
3. A closdd half-plane is the union of a half-plane and its boundary.

4. (a) closed (c) open
   (b) open (d) closed

5. (a)



(b)



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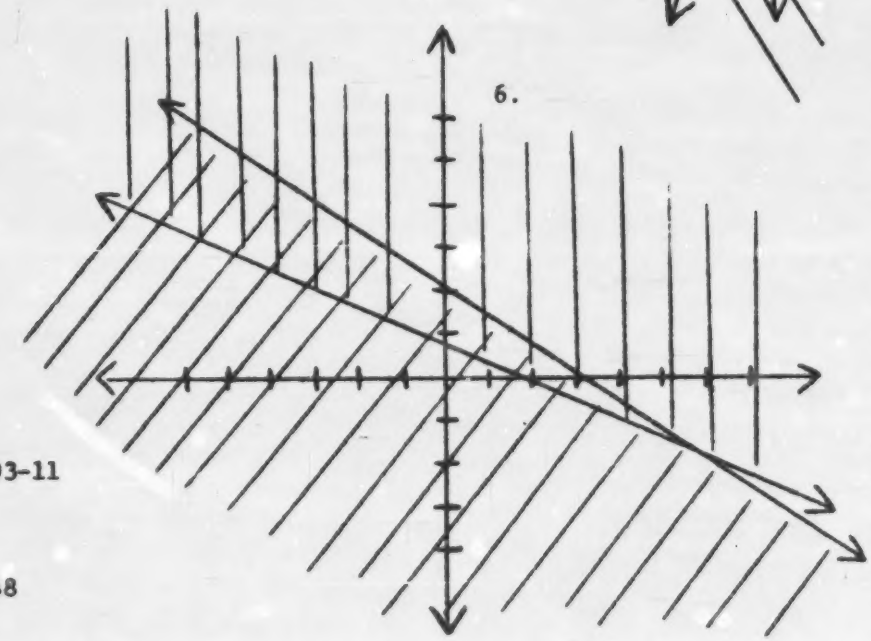

The End
Package 03-11